Arithmetic method - it also known as short cut method
Step 1. Which provides and easy way of finding optimal strategies of formal players in a pay off matrix. of size 2x2.

Step 1. The steps of this methods are as follow check the weather the matrix has a saddle point or not if not than deduct in each row the smaller pay off of largest and mark it outside the matrix.

Ques-

$$\begin{bmatrix} 5 & 1 \\ 3 & 4. \end{bmatrix} \begin{array}{l} (5-1) = 4 \text{ row wise} \\ (4-3) = 1 \end{array}$$

as no S.P

Step 2. deduct each column the smaller pay off from the largest and mark it below the matrix

$$\begin{array}{cc|c} \begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix} & & \begin{array}{c} 4 \\ 1 \end{array} \\ \begin{array}{cc} (5-3) & (4-1) \\ 2 & 3. \end{array} & & \end{array} \quad \text{column wise}$$

$$\begin{array}{cc} \begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix} & \begin{array}{c} 4 \\ 1 \end{array} \\ \begin{array}{cc} 2 & 3. \end{array} & \end{array}$$

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Step 3- enterchange each pair of substracting both of the row and column

$$\begin{array}{cc|c} \begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix} & & \begin{array}{c} 4 \\ 1 \end{array} \\ 2 \quad 3 & & \end{array} \longrightarrow \begin{array}{cc|c} \begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix} & & \begin{array}{c} 1 \\ 4 \end{array} \\ 3 \quad 2 & & \end{array}$$

Step4- Make them into fraction of the sum of the pair in each case

$$\begin{array}{cc} \begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix} & \begin{array}{c} 1/5 \\ 4/5 \end{array} \\ 3/5 \quad 2/5 & \end{array}$$

Step 5 Calculate the joint probability of each combination of X & Y strategies.

$$\begin{bmatrix} 5 \; \left(\frac{1}{5}\times\frac{3}{5}\right) & 1 \; \left(\frac{1}{5}\times\frac{2}{5}\right) \\ 3 \; \left(\frac{3}{5}\times\frac{4}{5}\right) & 4 \; \left(\frac{2}{5}\times\frac{4}{5}\right) \end{bmatrix}$$

$$5\times\frac{3}{25} + 1\times\frac{2}{25} + 3\times\frac{12}{25} + 4\times\frac{8}{25}$$

$$\frac{15+2+36+32}{25} = \frac{85}{25} = \frac{17}{5} = 3.4$$

Step 6- multiply each pay off by its joint probabilities and sum up to get the value of game

Qus-2 Find the optimal strategies of player A & B and value of game from following pay off matrix

$$\begin{array}{cc|cc} & & \text{Player B} & \\ & & B_1 & B_2 \\ \text{Player A} & A_1 & 20 & 5 \\ & A_2 & 10 & 15 \end{array}$$

$$\begin{array}{c|cc|c} & \text{Player B} & & \\ & B_1 & B_2 & \\ A_1 & 20 & 5 & 20-5 = 15 \\ A_2 & 10 & 15 & 10-15 = 5 \\ \text{column wise} & (20-10) & (15-5) & \\ & 10 & 10 & \end{array}$$

$$\begin{array}{cc|c} 20 & 5 & 5 \\ 10 & 15 & 15 \\ 10 & 10 & \end{array}$$

interchange

$$\begin{array}{cc|c} 20 & 5 & 5/20 \\ 10 & 15 & 15/20 \\ 10/20 & 10/20 & \end{array}$$

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$$\begin{bmatrix} 20 \times \frac{10}{20} \times \frac{5}{20} & 5 \times \frac{10}{20} \times \frac{5}{20} \\ 10 \times \frac{10}{20} \times \frac{15}{20} & 15 \times \frac{10}{20} \times \frac{15}{20} \end{bmatrix}$$

$$20 \times \frac{10}{20} \times \frac{5}{20} + 5 \times \frac{10}{20} \times \frac{5}{20} + 10 \times \frac{10}{20} \times \frac{15}{20} + 15 \times \frac{10}{20} \times \frac{15}{20}$$

$$\frac{1000 + 250 + 1500 + 2250}{20 \times 20}$$

$$\frac{5000}{400} = 12.5$$

Ques3-

$$\begin{array}{cc} & \begin{bmatrix} 10 & 13 & 16 \\ 5 & 12 & 14 \\ -2 & 13 & 10 \end{bmatrix} \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} \\ & \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \end{array}$$

Players

Principle of dominance

Since A_1 dominates A_3 so we can delete A_3

$$\begin{array}{c} \begin{bmatrix} 10 & 13 & 16 \\ 5 & 12 & 14 \end{bmatrix} \begin{array}{c} A_1 \\ A_2 \end{array} \\ \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \end{array}$$

$$\begin{array}{cc} \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{bmatrix} 10 & 13 \\ 5 & 12 \end{bmatrix} \\ & \begin{array}{cc} B_1 & B_2 \end{array} \end{array}$$

Since B_1 dominates B_3 we can delete B_3.

$$\begin{array}{c|cc|c} & B_1 & B_2 & \text{row wise} \\ \hline A_1 & 10 & 13 & 3 \\ A_2 & 5 & 12 & 7 \\ \hline \text{column wise} & 5 & 1 & \end{array}$$

interchange.–

$$\begin{array}{c|cc|c} & B_1 & B_2 & \\ \hline A_1 & 10 & 13 & 7 \\ A_2 & 5 & 12 & 3 \\ \hline & 1 & 5 & \end{array}$$

factorisation

$$\begin{array}{c|cc|c} & B_1 & B_2 & \\ \hline A_1 & 10 & 13 & 7/10 \\ A_2 & 5 & 12 & 3/10 \\ \hline & 1/6 & 5/6 & \end{array}$$

$$\begin{bmatrix} 10^{\,1/6 \times 7/10} & 13^{\,\frac{5}{6} \times 7/10} \\ 5^{\,1/6 \times 3/10} & 12^{\,\frac{7}{10} \times \frac{3}{10}} \end{bmatrix}$$

$$10 \times \frac{1}{6} \times \frac{7}{10} + 13 \times \frac{5}{6} \times \frac{7}{10} + 5 \times \frac{1}{6} \times \frac{3}{10} + 12 \times \frac{5}{10} \times \frac{3}{10}$$

$$\frac{70 + 455 + 15 + 180}{60}$$

$$\frac{720}{60} = 12.$$